

# Retardation of Particle Evaporation from Excited Nuclear Systems Due to Thermal Expansion

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## ABSTRACT

Particle evaporation rates from excited nuclear systems at equilibrium matter density are studied within the Harmonic-Interaction Fermi Gas Model (HIFGM) combined with Weisskopf's detailed balance approach. It is found that thermal expansion of a hot nucleus, as described quantitatively by HIFGM, leads to a significant retardation of particle emission, greatly extending the validity of Weisskopf's approach. The decay of such highly excited nuclei is strongly influenced by surface instabilities.

## I. INTRODUCTION

Studies of nuclear heavy-ion reactions at Fermi bombarding energies (a few tens of MeV per nucleon) have faced conceptual and experimental challenges. This is the domain in which the low-energy dissipative reaction mechanism [1] is expected to morph into one that is characterized by a merging of time scales for collective motion and relaxation processes and the population of a new phenomenological realm [2]. For example, for nuclear temperatures of  $T \sim 5\text{MeV}$ , nucleon evaporation times of the order of  $t_{\text{evap}} \approx 20\text{fm}/c$  have been estimated [2, 3]. Such short evaporation times describe a “prompt” nuclear decay occurring during the nuclear interaction between projectile and target, thus preventing a complete equilibration of the system. Furthermore, mechanical and chemical instabilities [4] expected for highly excited nuclear systems should exceed the range of conventional statistical models and cause novel nuclear decay modes [5, 6].

On the other hand, experimental observations [7, 8, 9] of the particle emission patterns in reactions at Fermi energies, induced by heavy projectiles, demonstrate the persistence of sequential decay of hot projectile-like fragments and their target-like reaction partners. Apparently, the primary hot fragments emit nucleons, light particles, and complex nuclear clusters with patterns characteristic of thermalized sources. While the presence of an additional, “intermediate-velocity source” component in the particle emission patterns (most likely, of dynamical origin) [10, 11, 12] tends to complicate the analysis, the existence of intense sequential evaporation components demonstrates unexpectedly high (meta-) stability of the primary hot projectile and target-like nuclei.

The present work investigates conditions for a consistency of the apparently long particle emission times with high nuclear excitations. It is shown that, consistent with the above observations, highly excited nuclear systems are indeed similar to a classical compound nucleus, once their expansion to the equilibrium matter density [13] is taken into account. The particle emission timescales are then long enough to allow for a meaningful equilibration of the system in the sense of Weisskopf’s detailed balance [14]. Relative decay rates for various statistical decay channels are governed by the entropy at a respective “transition” state, in which the state of the surface plays an important role. The transition state may be identified with an emission barrier for particle emission and

a separation saddle for statistical fission-like decay.

In the following section, the Weisskopf evaporation model is briefly reviewed. The effects of nuclear expansion are described in Section III, followed by a presentation of calculations in Sect. IV and brief conclusions (Sect. V).

## II. WEISSKOPF'S PARTICLE EVAPORATION TIME-SCALES

The evaporation time scales for (metastable) excited nuclear systems can be conveniently evaluated using Weisskopf's detailed balance [14] approach. This approach relates the probability  $W(E_A^*, \epsilon)$  for the emission of a particle with energy  $\epsilon$  by a compound nucleus with mass number  $A$  and excitation energy  $E_A^*$  to the cross section  $\sigma(\epsilon)$  for the reverse process of absorption of such a particle by a nucleus of mass number  $B$  and corresponding excitation energy  $E_B^*$ ,

$$W(E_A^*, \epsilon)d\epsilon = \sigma(\epsilon) \frac{gm\epsilon}{\pi^2 \hbar^3} \frac{\omega_B(E_B^*)}{\omega_A(E_A^*)} d\epsilon, \quad (1)$$

where  $\omega_A$  and  $\omega_B$  are the level densities of nuclei  $A$  and  $B$  at respective excitation energies,  $m$  is the mass of the emitted particle,  $\hbar$  is Planck's constant divided by  $2\pi$ , and  $g$  denotes the spin degeneracy.

In Eq. 1, excitation energies  $E_A^*$  and  $E_B^*$  of the compound nucleus  $A$  and the residue  $B$  are related via energy conservation

$$E_B^* = E_A^* - \epsilon - Q_{gg}, \quad (2)$$

where  $Q_{gg}$  is the  $Q$  - value for the particle separation from the ground state of nucleus  $A$ , leaving the residue  $B$  also in its ground state.

By expressing level densities  $\omega$  in terms of entropy  $S$

$$\omega_A(E_A^*) = e^{S_A(E_A^*)} \quad \text{and} \quad \omega_B(E_B^*) = e^{S_B(E_B^*)} \quad (3)$$

one rewrites Eq. 1 in a form

$$W(E_A^*, \epsilon)d\epsilon = \sigma(\epsilon) \frac{gm\epsilon}{\pi^2 \hbar^3} e^{S_B(E_A^* - Q_{gg} - \epsilon) - S_A(E_A^*)} d\epsilon, \quad (4)$$

which is well suited for use in conjunction with models such as HIFGM [13, 15]. Ultimately, the decay width  $\Gamma$  for a particular particle channel is obtained by integrating Eq. 4 over particle energy  $\epsilon$  and then by multiplying the result by  $\hbar$ , i.e.,

$$\Gamma(E_A^*) = \hbar \int_0^\infty W(E_A^*, \epsilon) d\epsilon = \frac{gm}{\pi^2 \hbar^2} \int_0^\infty \sigma(\epsilon) \epsilon e^{S_B(E_A^* - Q_{gg} - \epsilon) - S_A(E_A^*)} d\epsilon. \quad (5)$$

To be able to make use of Eq. 5 one needs additionally a prescription for calculating the cross section as a function of bombarding energy. In the context of Weisskopf's approach, one assumes customarily that the absorption cross section is equal to the geometrical cross section of the nucleus. This corresponds to a constant value of  $\sigma = \pi r_o^2$  for neutrons and an energy-dependent, Coulomb corrected expression for charged particles,

$$\sigma = \pi r_o^2 \left(1 - \frac{V_C}{\epsilon}\right), \quad (6)$$

where  $r_o$  is the nuclear radius parameter and  $V_C$  is the magnitude of the Coulomb barrier. Obviously, Eq. 6 is suited for both, charged particles and neutrons, with  $V_C = 0$  for the latter.

A useful approximation for the right-hand side of Eq. 5 can be obtained [14] for the neutron decay width. It assumes an identical functional dependence of the entropy on thermal excitation energy for both, mother and daughter nucleus, i.e.,  $S_A(x) = S_B(x)$ . Then, expanding the entropy  $S_B$  in a Taylor series about  $E_A^*$  to first order results in

$$S_B(E_A^* - Q_{gg} - \epsilon) - S_A(E_A^*) = -\frac{dS_A}{dE_A^*} * (Q_{gg} + \epsilon) = -\frac{1}{T_A} (Q_{gg} + \epsilon), \quad (7)$$

where use was made of the fact that the derivative of the entropy with respect to excitation energy,  $dS_A/dE_A^*$ , is equal to the inverse of the temperature,  $T_A$ , of nucleus A. Thus, for the neutron decay width one obtains an approximate expression

$$\Gamma_n(E_A^*) = \frac{gm}{\pi \hbar^2} r_o^2 T_A^2 e^{-\frac{Q_{gg}}{T_A}}, \quad (8)$$

revealing the strong temperature dependence of the decay width. It is this exponential temperature dependence that has raised concerns regarding the applicability of Weisskopf's equilibrium approach at higher excitation energies. Rationale given in Sect. III will demonstrate a much weaker effective temperature dependence of the decay width.

In figures shown further below, the decay rate is expressed in terms of the decay time scales parameter  $\tau$ , which is related to the decay width  $\Gamma$  via Heisenberg's uncertainty principle,  $\tau = \hbar/\Gamma$ .

### III. EFFECTS OF THERMAL EXPANSION OF NUCLEAR MATTER

Like most liquids, nuclear matter is expected to respond to thermal excitation by expanding, so as to arrive at a state of maximum entropy for any given total excitation energy. While thermal expansion should be predicted by any realistic model of bound nuclear matter, a particularly simple expression for the dependence of the asymptotic, equilibrium matter density on total excitation energy is provided by the Harmonic Interaction Fermi Gas Model (HIFGM). The essentials of this model [13, 15] are described below.

The central notion of the HIFGM is entropy. The relation between this entropy and the thermal excitation energy  $E_{therm}^*$  is assumed to be described by the regular Fermi-gas expression, such that

$$S(E_{tot}^*) = 2\sqrt{a(E_{tot}^* - E_{compr}^*)}, \quad (9)$$

where  $E_{tot}^*$  is the total excitation energy and  $E_{compr}^*$  is its collective compressional (potential energy) part. The difference between these latter two energies defines the random, thermal excitation  $E_{therm}^*$  and, hence, the entropy. Furthermore, the HIFGM adopts the matter density dependence of the level density parameter  $a$  (“little-a”) germane to the Fermi-gas model,

$$a = a_o \left(\frac{\rho}{\rho_o}\right)^{-\frac{2}{3}}, \quad (10)$$

where  $a_o$  is the ground-state value of the level-density parameter and  $\rho_o$  is the ground-state matter density. For simplicity, the HIFGM assumes a quadratic dependence of the compressional energy,  $E_{compr}^*$ , on the relative matter density [16]

$$E_{compr}^* = E_{bind} \left(1 - \frac{\rho}{\rho_o}\right)^2, \quad (11)$$

where  $E_{bind}$  is the ground-state binding energy of the system.

Given Eqs. 9-10, the equation for maximum entropy as a function of relative density can be resolved analytically, yielding [13]

$$\frac{\rho_{eq}}{\rho_o} = \frac{1}{4} \left( 1 + \sqrt{9 - \frac{E_{tot}^*}{E_{bind}}} \right). \quad (12)$$

Assessing qualitatively the effects of thermal expansion on the decay width (see Eq. 8), one notes that, according to Eq. 12, the matter density decreases with increasing total excitation energy. Consequently, the compressional part of the excitation energy and the level density parameter, both increase. *The latter two trends reduce the rate of increase of the system temperature with increasing excitation energy, as compared to that without expansion.* This follows from the Fermi-gas model relationship between temperature ( $T$ ), level-density parameter ( $a$ ), and thermal part ( $E_{therm}^*$ ) of the excitation energy,

$$E_{therm}^* = E_{tot}^* - E_{compr}^* = aT^2. \quad (13)$$

This effect can be viewed as a manifestation of the *LeChâtelier Principle*. The reduction in effective temperature from the value that it would have at normal matter density leads to a reduction of the decay width, as approximated by Eq. 8. This reduction is partially offset by the effects of *a*) an increased value of the inverse capture cross section ( $\sigma$ ) and *b*) a neutron binding energy ( $Q_{gg}$ ) that is reduced by an amount equal to the compressional energy per nucleon. Results of calculations performed using the “exact” Eq. 5 are discussed in the next section.

#### IV. RESULTS OF CALCULATIONS

Results of model calculations for the Weisskopf particle evaporation time scales are shown in Figs. 1 and 2. In Fig. 1, the evolution of the average nucleon evaporation time  $\tau$  with total excitation energy per nucleon, predicted for expanded Fermi matter at equilibrium density (solid line), is compared to that resulting for Fermi matter forced to stay at ground-state density (dashed line). As seen in this figure, if allowed to expand to equilibrium density, a nucleus excited to  $E_{tot}^*/A \geq 8 \text{ MeV}$  regains a degree of (meta)stability against nucleon evaporation comparable to that of a ground-state density nucleus at the much lower excitation of  $E_{tot}^*/A \sim 4.5 \text{ MeV}$ . The small increase in  $\tau$  predicted to occur near the high-excitation end of the curve is related to the negative heat capacity predicted by the HIFGM in that energy domain. Both these effects are

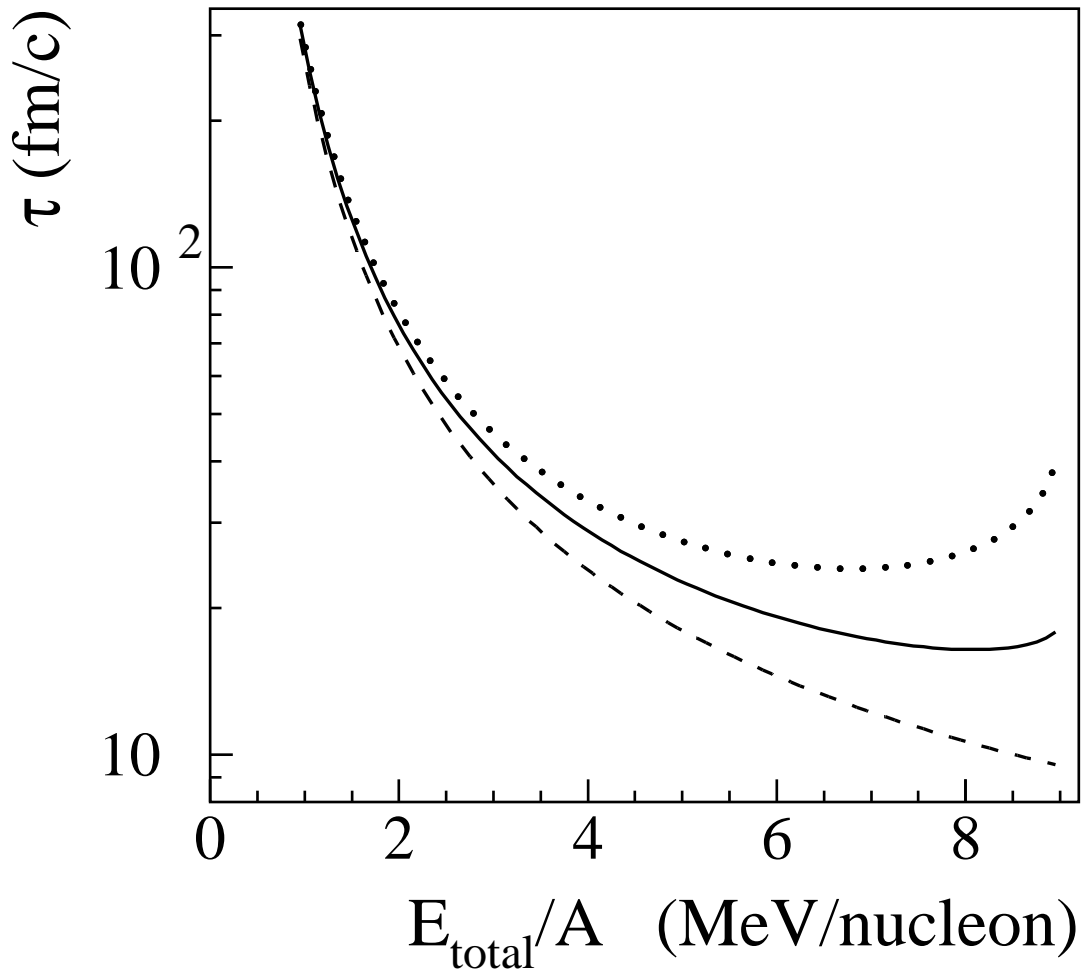


FIG. 1: Excitation-energy dependence of Weisskopf's nucleon evaporation time scales for nuclei at equilibrium (solid line) and ground-state (dashed line) matter density. The dotted line illustrates results of calculations with  $T = 0$  absorption cross section.

due to the fact that the considered maximum in entropy is not absolute, but conditional, obtained by imposing a certain functional form on the radial distribution of the nuclear matter density.

In Fig. 2, partial time scales are illustrated separately for neutron emission and for proton emission. In both cases, thermal expansion is seen to lead to a retardation of emission, extending the range of applicability of Weisskopf's approach.

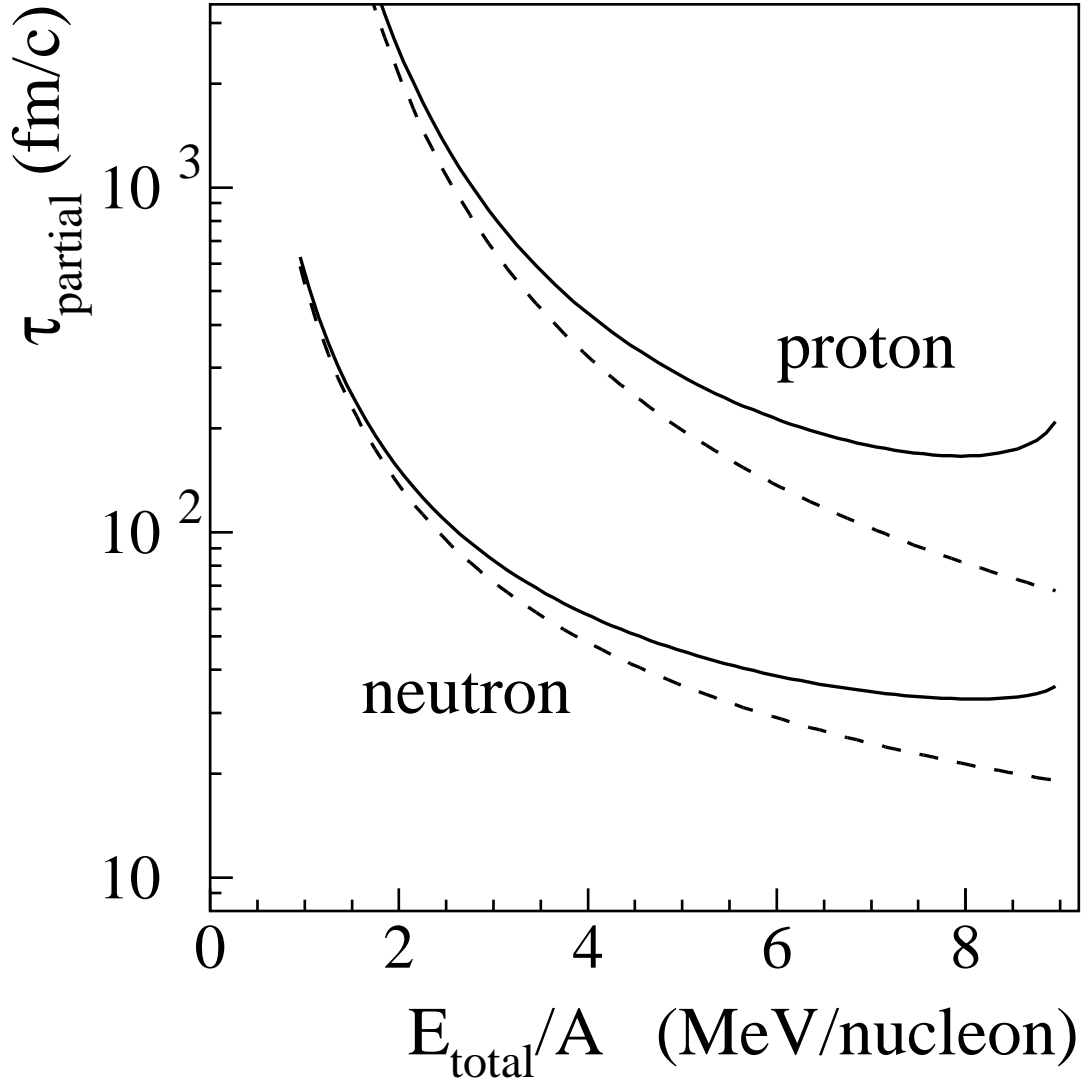


FIG. 2: Excitation-energy dependence of Weisskopf's neutron (bottom curves) and proton (top curves) partial evaporation time scales for nuclei at equilibrium (solid line) and ground-state (dashed line) matter density.

## V. CONCLUSIONS

The present calculations have shown that thermal expansion of a nucleus to equilibrium density has a stabilizing effect on the excited system, such that the most likely statistical decay channels are suppressed. This may give the system time to reach a more complete statistical equilibrium, where various surface modes are excited as well.



Such surface modes may be viewed as doorway states associated with cluster emission. It has been demonstrated elsewhere[13] that, when equilibrated at high excitation energies, such fission-like decay modes may compete successfully with nucleon emission. It should also be kept in mind that, with every neutron that escapes before these cluster emission modes are equilibrated, the probability for the emission of subsequent neutrons decreases, due to isospin effects, until it equals that for proton emission. This isospin effect adds additional credibility to the idea that Weisskopf's approach may be applicable at much higher nuclear excitation energies than commonly thought.

It is worth noting that, in its present schematic form, the HIFGM assumes that thermal expansion occurs in a self-similar fashion, i.e., that this expansion can be reduced to a rescaling of the radial coordinate. In addition, for the sake of simplicity, it is assumed that the effective nucleonic mass is equal to the free nucleon mass. A more complete modelling [17, 18] calls for more thorough and detailed treatment of the diffuse nuclear surface and its evolution with increasing excitation. In such a treatment, it would be highly desirable to account for the finite range of the effective nuclear interaction. This finite range may alter the thermostatic properties of surface matter with respect to those of bulk matter at equal density and temperature. Furthermore, it is necessary to include in a comprehensive modelling also thermal surface oscillations of the hot nuclear system, as the role of such oscillations is expected to increase [13] with increasing excitation energy (through decreasing surface tension or increasing surface entropy). It is to be expected that, in line with *LeChâtelier's Principle*, both, the bulk nuclear matter and the surface region will evolve in a concerted manner that increases entropy and stabilizes the overall system with respect to single nucleon emission even further.

## Acknowledgments

This work was supported by the U.S. Department of Energy grant No. DE-FG02-88ER40414.

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